A. LINEAR, BROWNIAN DYNAMIC MODEL

In this section, we modify the linear, Brownian dynamics theory of Van Lear and Uhlenbeck\(^1\) to account for filament tension and for various boundary conditions at the filament’s anchoring points. The filament is modeled as a beam with flexural rigidity and tension.

The displacement \(s(\zeta, t)\) of a vibrating beam with uniform flexural rigidity \(\kappa\) and tension \(\tau\) is given by the equation\(^2\)

\[
\rho \frac{\partial^2 s}{\partial t^2} + f \frac{\partial s}{\partial t} = \kappa \left( \frac{\partial}{L^2} \frac{\partial s}{\partial \zeta} \right) - \frac{\kappa}{L^4} \frac{\partial^4 s}{\partial \zeta^4} + F(\zeta, t),
\]

where \(t\) is time; \(0 < \zeta < 1\) is the dimensionless coordinate \((\zeta = x/L)\) that connects the two anchor points at the filament's ends and is scaled with the filament's length \(L\); \(\rho\) is the filament’s linear density (mass/length); \(f\) is the drag coefficient\(^3\); and \(F(\zeta, t)\) is a white noise forcing term. The forcing term has the properties

\[
\langle F(\zeta, t) \rangle = 0 \quad \text{(S2)}
\]

and

\[
\langle F(\zeta_1, t_1)F(\zeta_2, t_2) \rangle = \phi(\zeta_1 - \zeta_2, t_1 - t_2) \quad \text{(S3)}
\]
In the above, $\phi$ is approximated with the Dirac-Delta function, and $<a>$ denotes the ensemble average of $a$.

Following Van Lear and Uhlenbeck\(^1\), we seek solutions of the form

$$s(\zeta, t) = \sum_{n=1}^{\infty} X_n(\zeta)S_n(t),$$  \hspace{1cm} (S4)

where $X_n(\zeta)$ are orthonormal eigenvectors satisfying the homogeneous differential equation

$$pX_n'''' - \gamma X_n'''' + \lambda_n X_n = 0.$$  \hspace{1cm} (S5)

In the above, $p = \tau L^2/\rho \kappa$, $\gamma = 1/\rho$, and $\tau' = 0$ (uniform tension). Prime (') denotes a derivative with respect to $\zeta$. When the beam is simply supported, the boundary conditions consist of zero displacement and zero torque at the endpoints:

$$X(0) = X(1) = \frac{d^2}{d\zeta^2} X(0) = \frac{d^2}{d\zeta^2} X(1) = 0.$$  \hspace{1cm} (S6)

When the beam is clamped, the boundary conditions consist of zero displacement and zero angle (slope) at the endpoints:

$$X(0) = X(1) = \frac{d}{d\zeta} X(0) = \frac{d}{d\zeta} X(1) = 0.$$  \hspace{1cm} (S7)

Briefly, solving for the time part of the problem, computing the contributions of the various modes of the potential and kinetic energies, using the principle of equipartition of energy, assuming that the displacement and velocity are uncorrelated, and going to the asymptotic limit of $t \to \infty$, one obtains the expression

$$\langle s^2(\zeta) \rangle = \frac{k_B T}{\rho} \sum_{n=1}^{\infty} \frac{X_n^2(\zeta)}{\lambda_n}$$  \hspace{1cm} (S8)
for the variance. In the above, the eigenvalues $\lambda_n$ are functions of the filament's properties and the boundary conditions.

1. Simply Supported Case

In the case of a simply supported beam, the eigenfunctions $X_n(\zeta) = \sin(n\pi\zeta)$ and

$$\langle s^2(\zeta) \rangle = \sum_{n=1}^{\infty} \left( \frac{2k_B TL^3}{n^2 \pi^2 L^2 + \kappa n^4 \pi^4} \right).$$  \hspace{1cm} (S9)

The RHS of equation (9) can be summed up in a closed form:

$$\langle s^2(\zeta) \rangle = \frac{k_B TL}{2\tau} \left[ 2(1-\zeta)^2 + \frac{1}{L} \sqrt{\frac{\kappa}{\tau}} \left( -\coth \left( L \sqrt{\frac{\tau}{\kappa}} \right) - \cosh \left( (1-2\zeta)L \sqrt{\frac{\tau}{\kappa}} \right) \csc h \left( L \sqrt{\frac{\tau}{\kappa}} \right) \right) \right].$$  \hspace{1cm} (S10)

When $\kappa = 0$, equation (10) simplifies to the expression given by Van Lear and Uhlenbeck\(^1\) for a vibrating string ($\kappa = 0$):

$$\langle s^2(\zeta) \rangle = \frac{k_B TL}{\tau} \zeta(1-\zeta).$$  \hspace{1cm} (S11)

When we set $\tau = \nu^2$ and expand the hyperbolic functions into Taylor Series in terms of $\nu$ about $\nu = 0$, we find

$$\langle s^2(\zeta) \rangle = \frac{k_B TL}{2\nu} \left[ 2(1-\zeta)^2 + \frac{1}{L} \sqrt{\frac{\kappa}{\nu}} \left( -\frac{\sqrt{\kappa}}{L \nu} - \frac{L^3}{3 \sqrt{\kappa} \nu^3} + \frac{L^5}{45 \sqrt{\kappa} \nu^5} \right) \right].$$  \hspace{1cm} (S12)

In the limit of $\nu \rightarrow 0$, equation (12) simplifies to the expression for a simply supported beam undergoing bending alone ($\tau = 0$)\(^4\):

$$\langle s^2(\zeta) \rangle = \frac{k_B TL^3}{3\kappa} \zeta^2(1-\zeta)^2. \hspace{1cm} (S13)$$
2. Clamped Beam Case

In the case of clamped boundary conditions and $\tau = 0$, equation (S1) simplifies to the eigenvalue problem:

$$X(\zeta)^{''''} = m_n^4 X(\zeta),$$

(S14)

where $m_n^4 = \frac{\lambda_n L^4 \rho}{\kappa}$ are the eigenvalues of equation (14). The general solution has the form:

$$X_n(\zeta) = C_1 \sin(m_n \zeta) + C_2 \cos(m_n \zeta) + C_3 \sinh(m_n \zeta) + C_4 \cosh(m_n \zeta).$$

(S15)

After applying the Dirichlet and Neumann boundary conditions at $\zeta = 0$ and $\zeta = 1$, one obtains $C_2 = -C_4$, $C_1 = -C_3$, and

$$X_n(\zeta) = b_n \left[ a_n (\sin(m_n \zeta) - \sinh(m_n \zeta)) + (\cos(m_n \zeta) - \cosh(m_n \zeta)) \right],$$

(S16)

where

$$a_n = \frac{\cosh(m_n) - \cos(m_n)}{\sin(m_n) - \sinh(m_n)} = \frac{\sin(m_n) + \sinh(m_n)}{\cos(m_n) - \cosh(m_n)},$$

(S17)

and $b_n$ is determined by normalization. Equation (S17) can be simplified to

$$\cosh(m_n) \cos(m_n) = 1.$$  

(S18)

The eigenvalues $m_n \rightarrow \left(n + \frac{1}{2}\right)\pi$ as $n$ increases. By solving equation (S18), we obtain the eigenvalues of equation (S14). The first five eigenvalues when $\tau = 0$ are given in column 2 of Table 1. These values are, of course, identical to tabulated values$^5$.

Using equation (S8), we approximately have for the clamped case

$$\langle s^2(\zeta) \rangle = \frac{kTL^4}{\kappa} \sum_{n=1}^{N} \frac{1}{m_n^4} X_n^2(\zeta)$$

(S19)

or
\[
\langle s^2(\zeta) \rangle = \frac{kT L^3}{\kappa} \sum_{n=1}^{\infty} \frac{1}{m_n^2} \left[ \sin(m_n \zeta) + \sinh(m_n \zeta) \right] \left[ \cos(m_n \zeta) - \cosh(m_n \zeta) \right] \left[ \sin(m_n \zeta) - \sinh(m_n \zeta) \right] + \left[ \cos(m_n \zeta) - \cosh(m_n \zeta) \right].
\]  \hspace{1cm} (S20)

Witness that in (S19) and (S20), we approximated the infinite sum with a finite sum of \( N \) terms. The series (S19) and (S20) converge rapidly, like \( n^{-4} \) and, as we shall see shortly, just a few terms in the sum provide a reasonable approximation for the variance.

In general, the eigenvalues and eigenfunctions are functions of both \( \kappa \) and \( \tau \). When \( \tau \neq 0 \), we find it convenient to solve the problem with finite elements (COMSOL Multiphysics®). To verify the numerical code, we used it to reproduce the first five \( m_i \) values when \( \tau = 0 \) and \( \kappa = 7.3 \cdot 10^{-26} \) Nm². The numerically calculated values (Table 1, column 3) are in excellent agreement (within three significant digits) with the tabulated data\(^\text{5} \) (Table 1, column 2). As the series (equation (20)) converges rapidly, it was sufficient to use \( N = 5 \) to obtain results accurate within 1%. See Table 1.

Table 1: The first five eigenvalues as functions of \( \tau \). \( L = 7 \) \( \mu \)m and \( \kappa = 7.3 \cdot 10^{-26} \) Nm².

<table>
<thead>
<tr>
<th></th>
<th>( \tau = 0 ) (eq. (18))</th>
<th>( \tau = 0 )</th>
<th>( \tau = 1 ) pN</th>
<th>( \tau = 5 ) pN</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_1 )</td>
<td>4.730</td>
<td>4.7300</td>
<td>9.4378</td>
<td>13.756</td>
</tr>
<tr>
<td>( m_2 )</td>
<td>7.853</td>
<td>7.8532</td>
<td>13.490</td>
<td>19.497</td>
</tr>
<tr>
<td>( m_3 )</td>
<td>10.99</td>
<td>10.996</td>
<td>16.798</td>
<td>23.966</td>
</tr>
<tr>
<td>( m_4 )</td>
<td>14.14</td>
<td>14.138</td>
<td>19.816</td>
<td>27.811</td>
</tr>
<tr>
<td>( m_5 )</td>
<td>17.28</td>
<td>17.280</td>
<td>22.710</td>
<td>31.289</td>
</tr>
<tr>
<td>( \langle s^2(L/2) \rangle ), ( N = 5 )</td>
<td>0.1034</td>
<td>0.0061</td>
<td>0.0013</td>
<td></td>
</tr>
<tr>
<td>( \langle s^2(L/2) \rangle ), ( N = 10 )</td>
<td>0.1035</td>
<td>0.0062</td>
<td>0.0013</td>
<td></td>
</tr>
</tbody>
</table>
B. ESTIMATION OF THE FILAMENT’S TENSION

To estimate the filament’s tension $\tau$, we select $\tau$ such as to minimize the deviation between the measured and predicted variances.

To this end, we define the “penalty function”

$$\Theta(\kappa, \tau) = \int_0^1 \left( s_{\text{experiment}}^2(\zeta) - \left( s_{\text{theory}}^2(\zeta) \right) \right)^2 d\zeta.$$  \hspace{1cm} (S21)

We used the Matlab® optimization toolbox to find the "optimal" $\tau$ that minimizes equation (S21) when $\kappa$ is given or search for the “optimal” $\tau$ and $\kappa$ concurrently. The latter procedure proved not to be robust. Variants of this procedure are possible in which we sum up the deviations obtained in various experiments carried out under similar conditions.

![Diagram](image-url)

Figure S1: The optimization process used to estimate the tension being applied to actin filaments in an electric field.
Following an initial guess for $\tau$, the Matlab optimization toolbox repetitively called upon the finite element code to solve for the first five eigenfunctions, after which it calculated the penalty function (equation (S21)). The estimate for $\tau$ was corrected until a minimum for the penalty function was identified. The process is depicted schematically in Fig. S1. To illustrate that the process, indeed, yields a minimum for the penalty function, Fig. S2 depicts the penalty function (equation (S21)) as a function of $\tau$ when $\kappa$ is given ($E_{\text{rms}}=0.3 \, \text{V/\mu m}$). Witness the presence of a well-defined minimum. Attempts to concurrently estimate $\tau$ and $\kappa$ were not as successful. Fig. S3 depicts the penalty function as a function of both $\tau$ and $\kappa$ for the same data that was used for Fig. 2. The figure provides a reasonable estimate for $\tau$. The penalty function depends only weakly on the magnitude of $\kappa$. When we fix $\tau$ and vary $\kappa$, we find a shallow minimum, precluding us from obtaining a good estimate of $\kappa$.

Figure S2: The penalty function as a function of the tension $\tau$. $\kappa = 7.3 \cdot 10^{-26} \, \text{Nm}^2$, $E_{\text{rms}} = 0.3 \, \text{V/\mu m}$. 

- 7 -
Figure S3: The penalty function as a function of $\tau$ and $\kappa$.

$E_{\text{rms}} = 0.3 \text{ V/}\mu\text{m}$

C. REFERENCES


